POSSIBILITY FOR AN ALL-SUPERCONDUCTING
SYNCHRONOUS MOTOR OR GENERATOR*

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POSSIBILITY FOR AN ALL-SUPERCONDUCTING SYNCHRONOUS MOTOR OR GENERATOR*

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ABSTRACT

Possibilities for a motor or generator with superconducting armature and field winding are discussed, and the a-c losses that would be expected in the armature winding are calculated.

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INTRODUCTION

Attention has recently been focused on the development of synchronous motors and generators having superconducting rotating field windings,[1,2] the stator, or armature winding, being made of normal material. Since such a design obviously makes use of only half the potentiality of superconductivity, the possibility for a fully superconducting machine is considered here. The principal problem, of course, is the loss that occurs in superconductors under a-c operation. Such loss would occur in the stator winding due to the changing magnetic field, a problem which is not encountered in the rotor since, for normal operation in an ideal machine, the field in the rotor is constant. Because of this loss it has been assumed, heretofore, that a fully superconducting machine would be impractical. The problem is not that the superconductor losses are large, but, rather, the fact that a large amount of refrigerator power (typically about 500 to 1) is required to remove the heat at the low temperature necessary for superconductivity to exist. Nevertheless, considerable progress has been made in recent years in designing low-loss conductors, and the analysis here indicates that a fully superconducting machine is not beyond the

1. Applied Superconductivity Conference 1972, Annapolis, Maryland, IEEE Pub. No. 72CH 0682-5-TABSC.

2. IEEE Winter Power Conference, New York, 1973.

realm of feasibility. Such a machine would produce much larger torque

per unit volume than the "half-superconducting" machines presently under

consideration. In principal the increase in torque, or size reduction,

which could be obtained is proportional to the ratio of the current

densities in the superconducting and normal windings. However, since

this ratio is more than a power of ten, it is obvious that other factors

such as mechanical strength, stability, magnetic shielding, and heat

transfer will provide the practical limitations.

POWER DEVELOPED BY AN IDEAL MACHINE

Consider an idealized machine as shown by the cross-section

in Fig. 1, which consists of a stator and a rotor winding without iron.

In practice the machine would be shielded from its external surround-

ings by either an iron or an eddy current shield, but, for simplicity,

this feature is ignored. Between the radii R_1 and R_o on the rotor a

uniform current density flows along the axis with a similar current

density between the radii R_3 and R_2 on the stator. The angular dis-

tribution between poles of these current densities at an instant of

time is sinusoidal. The rotor and the field produced by the stator

currents rotate synchronously at the angular velocity ω_r. Cylindrical

coordinates on the stator are denoted by z, R and θ where θ is mea-

sured from a given X axis; and on the rotor by z, R, $\bar{\theta}$ where $\bar{\theta}$ is

measured from a symmetry axis on the rotor, $\bar{X}$. At time zero the angle

between these axes is ϕ. End effects along the machine axis, which is
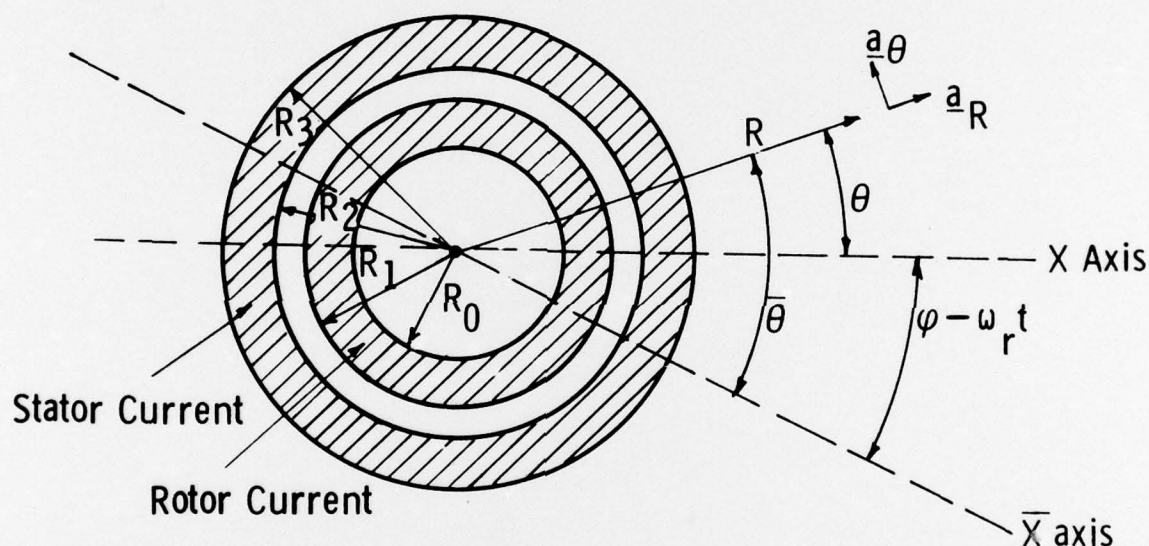
along the z direction, are ignored.

Fig. 1—Cross-section of machine about the axis of rotation. The X axis is an axis of reference on the stator, and $\overline{X}$ is a symmetry axis on the rotor. The angle between these axes is $\varphi - \omega_r t$ where t is the time and ω_r the angular velocity of the rotor. R and θ are cylindrical stator coordinates of a point, and R and $\overline{\theta}$ are cylindrical rotor coordinates. $\underline{a}_R$ and $\underline{a}_\theta$ are unit vectors at the point.

From the known expression for the magnetic field of a sinusoidal current sheet, it is easy, by integration over infinitesimal sheets, to obtain the magnetic fields produced by the current densities of Fig. 1. Mathematically, the current densities of the rotor and stator are

$$\mathbf{j}_r = j_{or} \sin p \, \bar{\theta} \, \mathbf{a}_z \qquad (1)$$

and

$$\mathbf{j}_s = j_{os} \sin (p \, \theta - \omega t) \, \mathbf{a}_z \qquad (2)$$

where ω is the angular frequency of the applied voltage, t is the time, p is the number of pole pairs, j_{or} and j_{os}, within the respective windings, are constants, and $\mathbf{a}_z$ is a unit vector in the z direction. The angular velocity of rotation ω_r is given by ω/p, and in terms of the stator coordinate $\bar{\theta} = \theta - \omega_r t + \phi$.

In the region of the rotor winding the magnetic field of the stator current, at time t = 0, is

$$\mathbf{H}_s = 2\pi j_{os} \left[\delta_{p1}(R_3 - R_2) + \delta_{p2} R \ln \frac{R_3}{R_2} + \frac{(1-\delta_{p1})(1-\delta_{p2})R^{p-1}}{(p-2)} \left(\frac{1}{R_2^{p-2}} - \frac{1}{R_3^{p-2}} \right) \right]$$

$$\times (\cos p\theta \mathbf{a}_R - \sin p\theta \mathbf{a}_\theta) \qquad (3)$$

where $\mathbf{a}_R$ and $\mathbf{a}_\theta$ are unit vectors in the direction of increasing R and θ, and δ is a Kronecker delta. The force per unit volume which this field produces on the rotor current in electromagnetic units is $\mathbf{j}_r \times \mathbf{H}_s$, and the torque about the z axis is $R(H_s)_R j_r$. Therefore the power P delivered by the machine per unit length along the axis is

$$P = \omega_r \pi (R_1^2 - R_0^2) \langle R(H_s)_R j_r \rangle, \tag{4}$$

where the angular brackets indicate an average over the volume of the rotor winding. It is convenient to rewrite (4) in the form

$$P = \frac{C}{2} \omega_r A_r A_s j_{or} j_{os} \sin p\phi \tag{5}$$

where A_r and A_s are the cross-sectional areas of the rotor and stator windings and

$$C = \frac{2\langle R(H_s)_R j_r \rangle}{\pi (R_3^2 - R_2^2) j_{or} j_{os} \sin p\phi} . \tag{6}$$

When the average is performed, C becomes

$$C = \frac{4}{3} \delta_{p1} \frac{(R_1^2 + R_0^2 + R_1 R_0)}{(R_1 + R_0)(R_3 + R_2)} + \delta_{p2} \frac{(R_1^2 + R_0^2)}{(R_3^2 - R_2^2)} \ell n \frac{R_3}{R_2}$$

$$+ \frac{4(1 - \delta_{p1})(1 - \delta_{p2})}{(p^2 - 4)} \frac{(R_1^{p+2} - R_0^{p+2})}{(R_1^2 - R_0^2)(R_3^2 - R_2^2)} \left(\frac{1}{R_2^{p-2}} - \frac{1}{R_3^{p-2}} \right). \tag{7}$$

For thin windings, where $(R_1 - R_0)/R_1 \ll 1$ and $(R_3 - R_2)/R_3 \ll 1$, the value of C reduces simply to $(R_0/R_2)^p$.

FIELD IN THE STATOR WINDING

Within the region of the stator winding the field produced by the stator current, at $t = 0$, is

$$H_s = 2\pi j_{os} \int_{R_2}^{R} \left(\frac{R'}{R}\right)^{p+1} dR' (\cos p\theta a_R + \sin p\theta a_\theta)$$

$$+ 2\pi j_{os} \int_{R}^{R_3} \left(\frac{R}{R'}\right)^{p-1} dR' (\cos p\theta a_R - \sin p\theta a_\theta) \tag{8}$$

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and the field produced by the rotor current is

$$\underset{\sim}{H}_r = \frac{2\pi j_{or}}{(p+2)} \frac{(R_1^{p+2}-R_o^{p+2})}{R^{p+1}} (\cos p\bar{\theta}\underset{\sim}{a}_R + \sin p\bar{\theta}\underset{\sim}{a}_\theta). \tag{9}$$

The sum of these two fields is

$$\underset{\sim}{H} = 2\pi\{j_{os}[f(R)+g(R)] \cos p\theta + j_{or}h(R) \cos p\bar{\theta}\}\underset{\sim}{a}_R$$

$$+ 2\pi\{j_{os}[f(R)-g(R)] \sin p\theta + j_{or}h(R) \sin p\bar{\theta}\}\underset{\sim}{a}_\theta \tag{10}$$

where

$$f(R) = \frac{1}{(p+2)} (R - \frac{R_2^{p+2}}{R^{p+1}}) \tag{11}$$

$$g(R) = \delta_{p1}(R_3-R) + \delta_{p2} R \ln \frac{R_3}{R} + \frac{(1-\delta_{p1})(1-\delta_{p2})}{(p-2)} (R-\frac{R^{p-1}}{R_3^{p-2}}) \tag{12}$$

and

$$h(R) = \frac{1}{(p+2)} \frac{(R_1^{p+2}-R_o^{p+2})}{R^{p+1}} . \tag{13}$$

The square of the magnitude of H is

$$H^2 = (2\pi j_{os})^2[f^2+g^2+2fg \cos 2p\theta] + (2\pi j_{or})^2h^2$$

$$+ 8\pi^2 j_{os}j_{or}[fh \cos p\phi + gh \cos p(2\theta + \phi)] \tag{14}$$

where $\theta+\phi$ has been substituted for $\bar{\theta}$.

The quantity of principal interest is $<H_m>$ where H_m is the maximum value of H as a function of θ, (which is the same as the maximum value of H at a point as a function of time), with the angular brackets indicating an average over R. This quantity determines the hysteresis

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loss in the stator winding, and will depend upon the power angle $p\phi$, in addition to the currents and winding dimensions.

LOSSES AND EFFICIENCY

If friction and windage are neglected, the only loss occurring in the machine is the a-c loss in the stator, since the field in the rotor is constant in time. It will be assumed that the stator winding is made up of closely packed turns of conductors which consist of a single wire, or a cable or braid of smaller wires. Each wire is, itself, a composite filamentary superconductor. If the wire diameter is very small compared with the thickness of the winding and the winding is properly distributed, the magnetic field is essentially that calculated in the previous section for the ideal case of a continuous current density.

In large transverse magnetic fields which completely penetrate the wire, the limiting loss in the winding, i.e., the minimum loss that can be obtained, is the hysteresis loss of the individual superconducting filaments. Assuming complete field penetration of each filament, this power loss per unit volume of wire is[3]

$$\frac{4}{3\pi^2} \, d\lambda j_c H_m \omega \tag{15}$$

where d is the filament diameter, j_c the critical current density, H_m the peak field as a function of time, and λ the fraction of superconductor in the wire. In addition, an eddy current loss occurs given by

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3. W. J. Carr, Jr., to be published.

$$\frac{\sigma H_m^2 \omega^2}{2} \frac{[(\frac{L}{2\pi})^2 + \frac{D^2}{16}]}{[1+(\frac{L}{2\pi\delta})^4]}, \tag{16}$$

and a hysteresis loss in the saturated outer layer of the wire (for w greater than the thickness of the first layer of filaments) given by[4]

$$\frac{2w}{D} (\frac{L}{2\pi\delta})^2 \frac{8H_m^2\omega}{27\pi^3} . \tag{17}$$

In these expressions σ is the conductivity in the wire transverse to the filaments, L is the twist length of the filaments, D is the diameter of the wire, δ is the skin depth given by

$$\delta^{-2} = 2\pi\sigma\ \omega, \tag{18}$$

and w is the maximum penetration of the current saturated layer, given by

$$w = \frac{H_m}{2\pi\lambda j_c} (\frac{L}{2\pi\delta})^2 [1+(\frac{\pi D}{L})^2]^{1/2} . \tag{19}$$

The problem in wire design is to make (16) and (17) small compared with (15). This is accomplished mathematically by letting D and L approach zero with L/D finite, however in practice limitations exist, since D must be large enough to carry the required current, and L is limited to

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4. For an appreciable transport current the effect of the transport current on the losses must be included. The effect will be small if the ratio of the transport current to the critical current is small.

about six times the wire diameter.[5-6] When the loss criterion cannot
be met with a single wire, a technique suggested by many authors is to
choose D small enough so a suitably small L may be obtained, with the
desired conductor being formed from a cable or braid of such wires,
insulated or semi-insulated from one another. This technique may be
feasible, in particular, if only a small number of wires are required,
and the wires are well-transposed and well-exposed to the liquid helium.

If the total loss per unit volume is written as k times (15),
then the power loss in the stator per unit axial length is

$$P_L = \frac{A_s 4k}{3\pi^2} \, d \, \alpha \, j_c \langle H_m \rangle \omega \tag{20}$$

where α is the fraction of superconductor in the volume occupied by the
stator winding and $\langle H_m \rangle$ is again the average of H_m over R, for the
stator winding. If the refrigerator power necessary to remove the heat
loss is βP_L, than with the use of (5) the fractional loss is

$$\frac{\beta P_L}{P} = \frac{8\beta k \alpha p d j_c \langle H_m \rangle}{3\pi^2 C \, A_r \, j_{or} j_{os} \, \sin p\phi}. \tag{21}$$

For a given machine angle $p\phi$, this ratio depends upon the
number of pole pairs through the factor $p\langle H_m \rangle / C$. In the thin winding
approximation this factor is proportional to $p(R_2/R_o)^p$, $\langle H_m \rangle$ being
independent of p, and it follows that the ratio (21) is smallest for

5. M. N. Wilson, Proc. 1972 Applied Superconductivity Conf. Annapolis,
   Md., 385, IEEE, NY.

6. W. Gilbert, F. Voelker, R. Acker and J. Kaugerts, Proc. 1972
   Applied Superconductivity Conf. Annapolis, Md., 486, IEEE, NY.

a single pole pair. In the latter case for thin windings

$$\frac{\beta P_L}{P} = \frac{8}{3\pi^2} \frac{R_2 \beta k \alpha d j_c}{R_o A_r j_{or} j_{os}} \frac{<H_m>}{\sin \phi}, \tag{22}$$

and if $<H_m>$ is "normalized" in terms of the field H_r in the gap due
to the rotor winding, where from (9)

$$H_r \approx 2\pi j_{or}(R_1 - R_o) = \frac{2A_r j_{or}}{R_1 + R_o}, \tag{23}$$

$$\frac{\beta P_L}{P} = \frac{16 \ \beta k \ d}{3\pi^2 (R_1 + R_o)\sin \ \phi} \ (\frac{R_2}{R_o})(\frac{\alpha j_c}{j_{os}})(\frac{<H_m>}{H_r}). \tag{24}$$

The value of j_{os} for normal operation must be small enough
such that on overload it does not exceed αj_c. If $\alpha j_c / j_{os}$ is taken to be
5, and $<H_m>/H_r$ and R_2/R_o are approximated by unity, then for the assumed
values $\beta = 400$, $d = 5 \times 10^{-4}$ cm, $\sin \phi = 0.7$ and[7] $k = 1.5$

$$\frac{\beta P_L}{P} = \frac{0.6}{R_1} \tag{25}$$

which leads to 98% efficiency for $R_1 = 30$ cm.

No allowance has been made here for the loss due to heat leak
from the leads. If this loss is assumed to be 2×10^{-3} watts per ampere,
and a β of 10^3 is assumed, the fractional loss is about 2/V where V
is the voltage applied to the winding.

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7. For example, for a frequency of 3 Hertz and $\delta = 0.8$ cm, the eddy
    current loss is about 50% of the hysteresis loss, and (17) is
    negligible if D = 0.02, $2\pi\lambda j_c = 10^5$ ab amps/cm^2, L/D = 6, d = 5 x 10^{-4}
    and $H_m = 10^4$ Oe.

CONCLUSIONS

It is feasible to construct a complete superconducting rotating synchronous machine having an overall efficiency comparable with the efficiency for an ordinary machine. The extent to which the potential for higher unit torque and smaller size may be realized depends upon factors such as the limiting magnetic field for which a thermally stable stator winding may be designed, the strength of materials available for holding together the rotor and stator windings, and the size of the magnetic shield. While some of these limits can be reasonably well calculated, determination of the limit imposed by conductor design requires a program of testing. Any advantage from size reduction of the machine would be partially offset by the necessity of having a helium refrigerator, with an imput power equal to a few per cent of the machine rating. Since the two may be separately located, however, for some applications this disadvantage would not be overriding.